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Oscillating Supersonic/Hypersonic Wings at High Incidence

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An approximate analytic method is developed for predicting the aerodynamic stability of oscillating supersonic/hypersonic flat wings at a mean angle of attack. It uses the known exact unsteady unified supersonic/hypersonic flow solution for a two-dimensional flat plate plus the strip theory approximation to obtain the formulas for the stability derivatives. They are applicable for wings of arbitrary planform shape at arbitrary angles of attack provided the shock wave is attached to the leading edge of the wing. Good agreement is obtained with existing theories in various special cases. The formulas for stability derivatives given here become exact in the Newtonian limit.

I. Introduction

UNSTEADY supersonic/hypersonic aerodynamics has been studied extensively using potential flow theory and hypersonic small-disturbance theory. While the former theory holds for moderate supersonic Mach number, the latter for hypersonic Mach number, and both for small angles of attack only, there is evidently a need for a unified supersonic/hypersonic flow theory that is applicable for large as well as small angles of attack.

For two-dimensional flow, exact solutions were given by Carrier[1] and Hui[2] for the case of an oscillating wedge and by Hui[3] for an oscillating flat plate. They are valid uniformly for all supersonic Mach numbers and for arbitrary angles of attack or wedge angles, provided that the shock waves are attached to the leading edge of the body.

For an oscillating triangular wing in supersonic/hypersonic flow, the shock wave may be attached to, or detached from, the leading edges, depending on the combination of the flight Mach number M_∞, the angle of attack α, the ratio of the specific heats γ of the gas, and the swept-back angle of the wing. The attached-shock case was studied by Liu and Hui[4] (for the lower surface flow only), whereas the detached-shock case in hypersonic flow was studied by Hui and Hemdan[5]; both are valid for general angles of attack. The analytical results of these authors show that there is only a slight dependence of the stability derivatives on the aspect ratio of the triangular wing, except near shock detachment. This would seem to suggest that a local two-dimensional approach to a three-dimensional problem may, in certain situations, yield useful approximations.

In this paper, we apply the strip theory to study the problem of stability of an oscillating flat plate wing of arbitrary planform shape placed at a certain mean angle of attack in a supersonic/hypersonic stream. Thus, at each spanwise station, the flow will be assumed locally two-dimensional with a shock wave attached to the leading edge for which the exact two-dimensional solutions for a flat plate[2,3] will be used. In this way, the stability derivatives of a wing of arbitrary planform are obtained analytically in closed form. The nature of this strip theory suggests that it will not be valid for detached-shock flows as it would be approximating the elliptic cross flow, which prevails under the situation, by a hyperbolic cross flow. The mean angle of attack would, by nature of inviscid theory, also exclude the range in which vortex roll-ups and/or flow separation became significant. However, it should be expected to give a fair approximation to attached-shock flows with only supersonic leading and trailing edges. Thus, for a given wing planform at a given angle of attack, the accuracy of the strip theory in approximating the actual three-dimensional flow around the wing is expected to increase with increasing flight Mach number. Indeed, the strip theory becomes exact in the Newtonian limit since the Newtonian flow, in which fluid particles do not interact with each other, is truly two-dimensional locally.

In Sec. II, the results of the exact two-dimensional supersonic flow over an oscillating flat plate will be recapitulated which will then be used in Sec. III to derive the formulas for the stability derivatives of an oscillating wing of general planform at arbitrary angles of attack. Comparisons of the strip theory in several special cases with various existing theories will be presented in Secs. IV-VI, and conclusions in Sec. VII.

II. Surface Pressure on a Two-Dimensional Oscillating Flat Plate

Consider small-amplitude, slow-pitching oscillation of a three-dimensional flat plate wing of arbitrary planform shape placed at a mean angle of attack α in a supersonic/hypersonic stream U_∞. Let the pitching motion be described by the pitch angle θ

$$\theta(t) = \bar{\theta} e^{i\omega t} \tag{1}$$

where t is the time variable, $\bar{\theta}$ the amplitude, and ω the frequency of the oscillation. The reduced frequency k is defined as

$$k = \omega \ell / U_\infty \tag{2}$$

where ℓ is the root chord of the wing. We shall assume that $\bar{\theta}$, $k \ll 1$ and terms $0(\bar{\theta}^2, \bar{\theta}k^2)$ and higher will be neglected.§

In this section, we consider the special case of a two-dimensional flat plate wing of infinite span with attached shock waves (Fig. 1), and we shall give the surface pressure coefficient C_p, which is defined as usual by

$$C_p \equiv (p - p_\infty) / ½\rho_\infty U_\infty^2 \tag{3}$$

where p_∞ and ρ_∞ are freestream pressure and density, and p the instantaneous pressure. Let the pivot axis be at a distance

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§The analysis could be extended to include higher-order frequency terms.

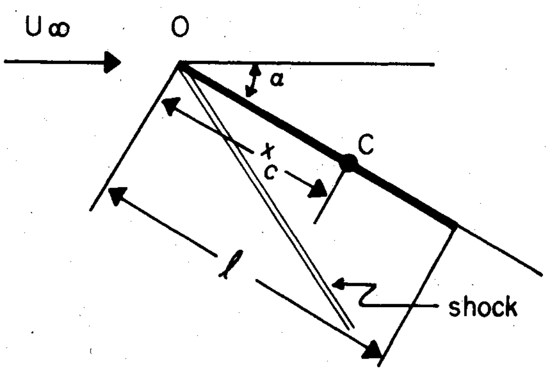

Fig. 1 Oscillating flat plate.

x_c from the leading edge; then the surface pressure is necessarily of the form[2,3]

$$C_p(x,t) = (C_p)_0 + \theta(t)[A + (i\omega/U_\infty)(Bx - Cx_c)] \tag{4}$$

where the coordinate x is measured from the leading edge along the chord, and the dimensionless constants A, B, and C are functions of M_∞, α, and γ. In Eq. (4), $(C_p)_0 = (C_p)_{\theta\equiv 0}$ is the steady pressure at the mean angle of attack α; it has no contributions to the stiffness derivatives or the damping-in-pitch derivative and will not be considered further in this paper.

A. Lower Surface

The flow over the lower surface of an oscillating two-dimensional flat plate at a mean angle of attack α is the same as that over an oscillating wedge of semiwedge angle α. The latter flow was given exactly in Ref. 2, from which we get

$$A = \lambda_0 \tilde{C}/\tilde{A} \equiv A_\ell \tag{5a}$$

$$B = \mu_0(2G - I) \equiv B_\ell \tag{5b}$$

$$C = \mu_0 I \equiv C_\ell \tag{5c}$$

where

$$\lambda_0 = 2\rho_0 u_0^2 / M_0 \rho_\infty U_\infty^2 \qquad \mu_0 = 2\rho_0 u_0 / M_0 \rho_\infty U_\infty \tag{6}$$

M_0 is the Mach number, ρ_0 the density, and u_0 the flow velocity in the two-dimensional steady flow (when $\theta \equiv 0$) over the wedge, the constants $\tilde{A}$, $\tilde{C}$, G, and I are all given in Ref. 2.¶

B. Upper Surface

The upper surface flow is an unsteady Prandtl-Meyer flow, and may be obtained from Ref. 3 by extending it to the case of arbitrary pivot position. Thus, for the same pitch angle θ as the lower surface, we get

$$A = -\lambda_1 M_1 / (M_1^2 - 1)^{1/2} \equiv A_u \tag{7a}$$

$$B = -\mu_1 M_1 (M_1^2 - 2)/(M_1^2 - 1)^{3/2} \equiv B_u \tag{7b}$$

$$C = -\mu_1 M_1 / (M_1^2 - 1)^{1/2} \equiv C_u \tag{7c}$$

where

$$\lambda_1 = 2\rho_1 u_1^2 / M_1 \rho_\infty U_\infty^2 \qquad \mu_1 = 2\rho_1 u_1 / M_1 \rho_\infty U_\infty \tag{8}$$

M_1 is the Mach number, ρ_1 the density, and u_1 the flow velocity in the steady Prandtl-Meyer flow past the flat plate at a constant deflection angle α.

¶To correct a misprint in Ref. 2, $\tilde{C}$ of Ref. 2 should be multiplied by a factor of 2.

When both the upper and lower surface flows are taken into account, we have

$$A = A_\ell - A_u, \quad B = B_\ell - B_u, \quad C = C_\ell - C_u \tag{9}$$

III. Stability Derivatives of a Three-Dimensional Wing

Consider now a three-dimensional flat plate wing of arbitrary planform shape (Fig. 2) oscillating about the pivot axis CC around a mean angle of attack α in a uniform supersonic/hypersonic flow. The Oxy plane is taken to be the wing plane at its mean position. Let ℓ denote the root-chord and b the semispan of the wing. The equations for the leading and trailing edge of the wing are given by

$$x/\ell = f(y/b) \tag{10a}$$

$$x/\ell = g(y/b) \tag{10b}$$

Obviously,

$$g(0) - f(0) = 1 \tag{11}$$

A. Surface Pressure

We shall use the strip theory to find unsteady pressure on the wing surface. The theory assumes that the flow over a chordwise strip LT is locally two-dimensional, i.e., it is the same as the supersonic flow of the same Mach number past a flat plate of infinite span, of which LT is a cross section. The pressure at a point $P(x,y)$ on the strip can therefore be calculated using Eqs. (4) and (9), in which the coordinates x and x_c are to be measured from the local leading-edge position L. Thus, with the changes $x \to x - \ell f$ and $x_c \to x_c - \ell f$ in Eq. (4), we get the surface pressure coefficient at a point $P(x,y)$ on the three-dimensional wing.

$$C_p(x,y,t) = \theta(t)\left\{A + ik\left[B\frac{x}{\ell} + (C-B)f\left(\frac{y}{b}\right) - C\frac{x_c}{\ell}\right]\right\} \tag{12}$$

B. Stability Derivatives

The pitching moment coefficient C_m, the stiffness derivative $-C_{m\theta}$, and the damping-in-pitch derivative $-C_{m\dot{\theta}}$ are defined as usual by

$$C_m = \frac{M}{\frac{1}{2}\rho_\infty U_\infty^2 \ell S} = \frac{1}{\ell S}\iint_S (x - x_c) C_p(x,y,t)\,\mathrm{d}S$$

$$\equiv \theta(t)[(-C_{m\theta}) + ik(-C_{m\dot{\theta}})] \tag{13}$$

where M is the moment of the unsteady pressure force about the pivot axis CC, and S is the planform area of the wing.

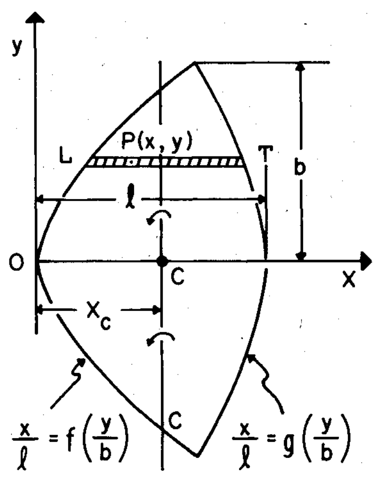

Fig. 2 Wing planform geometry.

Using Eq. (12) in Eq. (13) and carrying out the chordwise integrations, we get

$$-C_{m_\theta}=A[I_1-(x_c/\ell)] \tag{14a}$$

$$-C_{m_{\dot\theta}}=[BI_2+(C-B)I_4]-[(B+C)I_1+(C-B)I_3]x_c/\ell+C(x_c/\ell)^2 \tag{14b}$$

where

$$I_1=\frac{\ell b}{S}\int_0^1[g^2(\eta)-f^2(\eta)]\mathrm{d}\eta \tag{15a}$$

$$I_2=\frac{2\ell b}{3S}\int_0^1[g^3(\eta)-f^3(\eta)]\mathrm{d}\eta \tag{15b}$$

$$I_3=\frac{2\ell b}{S}\int_0^1 f(\eta)[g(\eta)-f(\eta)]\mathrm{d}\eta \tag{15c}$$

$$I_4=\frac{\ell b}{S}\int_0^1 f(\eta)[g^2(\eta)-f^2(\eta)]\mathrm{d}\eta \tag{15d}$$

For the stability derivatives, Eqs. (14) apply to flat wings of arbitrary planform shape at arbitrary angle of attack in supersonic/hypersonic flow, provided the shock wave is attached to the leading edge of the wing. In the following sections we shall discuss some useful special cases and compare them to the existing theories.

IV. Newtonian Limit

In the Newtonian limit as $M_\infty\rightarrow\infty$ and $\gamma\rightarrow 1$ independently, the constants A, B, and C reduce to [Eqs. (16) are also obtainable directly from Hui and Tobak[6]]

$$A_u=B_u=C_u=0$$

$$A=A_\ell=4\sin\alpha\cos\alpha$$

$$B=B_\ell=8\sin\alpha$$

$$C=C_\ell=4\sin\alpha \tag{16}$$

where

$$-C_{m_\theta}=2\sin2\alpha[I_1-(x_c/\ell)] \tag{17a}$$

$$-C_{m_{\dot\theta}}=4\sin\alpha[(2I_2-I_4)-(3I_1-I_3)x_c/\ell+(x_c/\ell)^2] \tag{17b}$$

It should be pointed out that the strip theory in the Newtonian limit is actually exact and involves no approximations since the Newtonian flow, in which fluid particles do not interact with each other, is truly two-dimensional locally over the flat plate wing. Therefore, Eqs. (17) represent exact formulas for the stability derivatives of flat plate wings of arbitrary planform shape in the Newtonian limit.

For delta wings with power-law leading edge, i.e.,

$$f(y/b)=(y/b)^{1/n} \tag{18a}$$

$$g(y/b)=1 \tag{18b}$$

we have

$$S=2\ell b/(n+1) \tag{19a}$$

$$I_1=(n+1)/(n+2) \tag{19b}$$

$$I_2=(n+1)/(n+3) \tag{19c}$$

$$I_3=n/(n+2) \tag{19d}$$

$$I_4=n/(n+3) \tag{19e}$$

Hence

$$-C_{m_\theta}=2\sin2\alpha\left[\frac{n+1}{n+2}-\frac{x_c}{\ell}\right] \tag{20a}$$

$$-C_{m_{\dot\theta}}=4\sin\alpha\left[\frac{n+2}{n+3}-\frac{2n+3}{n+2}\frac{x_c}{\ell}+\left(\frac{x_c}{\ell}\right)^2\right] \tag{20b}$$

The damping-in-pitch derivative Eq. (20b) is a quadratic in the pivot axis position x_c and has a minimum value

$$(-C_{m_{\dot\theta}})_{\min}=\left[\frac{4(n+2)}{n+3}-\left(\frac{2n+3}{n+2}\right)^2\right]\sin\alpha \tag{21}$$

whose variation with power n is plotted in Fig. 3. It is seen that $(-C_{m_{\dot\theta}})_{\min}$ decreases with increasing power n. We may therefore conclude that leading-edge concaveness of a flat delta wing has the effect of decreasing its dynamic stability, whereas leading-edge convexness tends to increase dynamic stability according to Newtonian flow theory. However, since the minimum damping is always positive, we also conclude that pitching oscillating flat delta wings with power-law leading edge in Newtonian flow are always dynamically stable. This is in direct contrast to pitching oscillating airfoils in Newtonian flow[6] which can become dynamically unstable if the profile of the airfoil is sufficiently concave.

In the special case of a triangular delta wing, $n=1$, and Eqs. (20) reduce to

$$-C_{m_\theta}=2\sin2\alpha\left[\frac{2}{3}-\frac{x_c}{\ell}\right] \tag{22a}$$

$$-C_{m_{\dot\theta}}=4\sin\alpha\left[\frac{3}{4}-\frac{5}{3}\frac{x_c}{\ell}+\left(\frac{x_c}{\ell}\right)^2\right] \tag{22b}$$

Equations (22) may be compared to the more sophisticated thin shock-layer theory of Hui and Hemdan[5] for an oscillating slender triangular delta wing with detached shock wave at high angle of attack. Although the Newtonian flow corresponds to the attached-shock case, it is interesting to see that Eq. (22a) for the stiffness derivative is identical with that of the thin shock-layer theory [Ref. 5, Eq. (29)] when $0(k^2)$ terms are neglected. This suggests that the in-phase component and hence the stiffness derivative in high Mach number flow are insensitive to the shock wave being attached to, or detached from, the wing's leading edges.

What is more interesting is that when the thin shock-layer theory for detached shock flow is extended to shock attachment, i.e., as $\Omega\rightarrow 2$ and hence $G(\Omega)\rightarrow 1/2$ in Fig. 2 of Ref. 5, the damping-in-pitch derivative, Eq. (30) of Ref. 5, becomes identical to the Newtonian theory, Eq. (22b) above. The value of the sophisticated gasdynamic thin shock-layer theory for a triangular wing is then to confirm the validity of

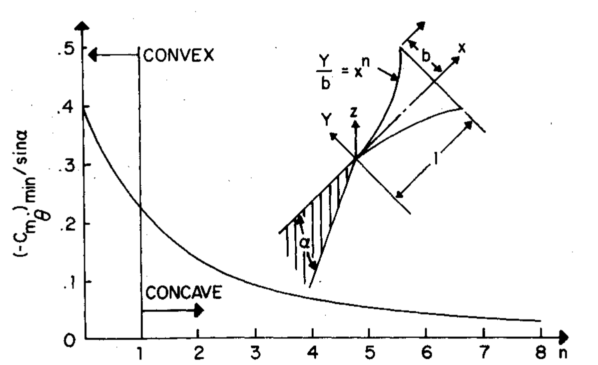

Fig. 3 Variation with power n of minimum damping-in-pitch derivative for delta wing with power-law leading edge in Newtonian flow.

the unsteady Newtonian flow theory for large but finite Mach number flow; however, the latter holds for wings of arbitrary planform shape at arbitrary angles of attack.

V. Delta Wings with Power-Law Leading Edge

We now consider the case of an oscillating delta wing with power-law leading edge in supersonic/hypersonic flow. In this case, the formulas for the stability derivatives, after using Eqs. (18) and (19), simplify to

$$-C_{m_\theta} = A\left[\frac{n+1}{n+2} - \frac{x_c}{\ell}\right] \tag{23a}$$

$$-C_{m_{\dot\theta}} = \frac{B+nC}{n+3} - \frac{B+(2n+1)C}{n+2}\frac{x_c}{\ell} + C\left(\frac{x_c}{\ell}\right)^2 \tag{23b}$$

For the special case of a triangular wing, i.e., for $n=1$, Eqs. (23) are compared with the more sophisticated gasdynamic theory of Liu and Hui[4] in Fig. 4 for the case $M_\infty=4$, $\alpha=15$ deg, $\gamma=1.4$ and the swept-back angle $\chi=50$ deg. Fair agreement is seen for the stiffness derivative; a greater difference occurs for the damping derivative. Comparisons for the variation of the damping-in-pitch derivative for flight Mach numbers greater than 3 are given in Fig. 5 for several values of the swept-back angle. It is a consequence of the strip theory approximation that the stability derivatives of a triangular wing are independent of its swept-back angle χ. Nevertheless, the agreement of the strip theory with that of Liu and Hui is seen to be good for all χ, but deteriorates with decreasing Mach number.

For a general power-law delta wing, the stability derivatives depend on the power n. As $n \to 0$, the delta wing becomes a two-dimensional flat plate and Eqs. (23) tend to the corresponding formulas for a flat plate,[7] as they should.

The stability of a delta wing may be investigated from Eq. (23b). Thus, for a given power n, the damping-in-pitch derivative reaches a minimum value

$$(-C_{m_{\dot\theta}})_{\min} = -\frac{1}{C}\left[\frac{B+(2n+1)C}{n+2}\right]^2 + \frac{4(B+nC)}{n+3} \tag{24}$$

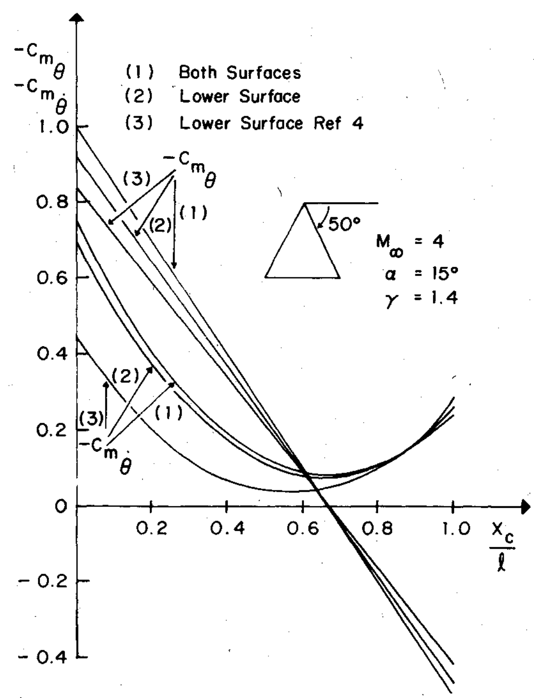

Fig. 4 Comparison of stability derivatives with theory of Liu and Hui for a triangular wing.

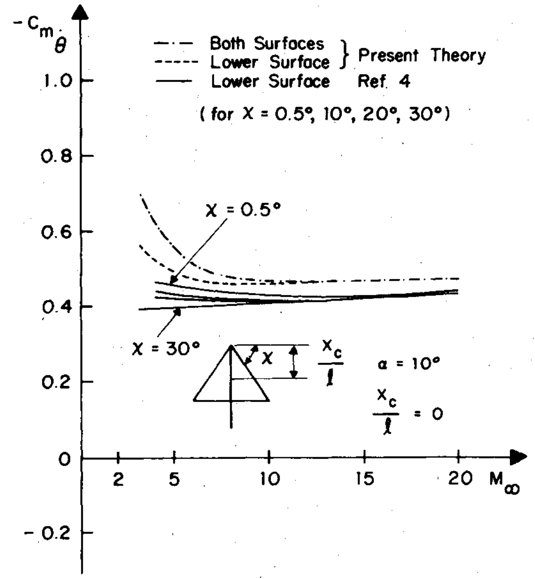

Fig. 5 Comparisons of damping-in-pitch derivatives.

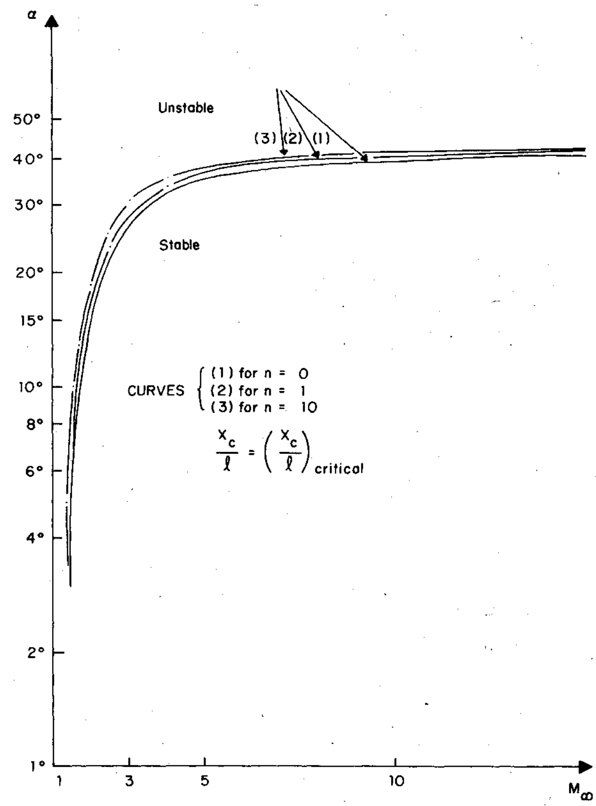

Fig. 6 Stability boundaries of power-lawed delta wings.

which occurs at the critical pivot axis position

$$\frac{x_c}{\ell} = \frac{1}{2C}\frac{B+(2n+1)C}{n+2} \equiv \left(\frac{x_c}{\ell}\right)_{\mathrm{cr}} \tag{25}$$

A stability boundary may be given for a delta wing n by equating $(-C_{m_{\dot\theta}})_{\min}=0$. These are plotted in Fig. 6 as the angle of attack α vs the flight Mach number M_∞ for $\gamma=1.4$ and for several values of the power n. For (α, M_∞) corresponding to a point below the curve, the pitching motion of the wing is dynamically stable, whereas for (α, M_∞) corresponding to a point above the curve, the pitching motion of the wing is dynamically unstable for a range of pivot axis position around $(x_c/\ell)_{\mathrm{cr}}$. It is seen from Fig. 6 that all of the delta wings with power-law leading edges have practically the same stability boundary as that for a flat plate $n=0$. However, the values of the damping-in-pitch derivatives for situations not corresponding to neutral damping do vary significantly with the power n. An example is shown in Fig. 7 for Mach number $M_\infty = 4$.

As remarked in the Introduction, the strip theory given here is expected to hold only for the flight conditions such that the shock wave is attached to the leading edge. The preceding conclusions regarding the effects of the power n of a power-law delta wing on its dynamic stability must be interpreted within these flight conditions.

VI. Comparisons with Potential Flow Theory

Consider the case of an oscillating swept-back tapered wing with straight supersonic leading and trailing edges and streamwise tips (Fig. 8). In this case, Eqs. (15) become

$$I_1 = \frac{\ell b}{S}\left(\frac{K_2^2 - K_1^2}{3} + K_2 + 1\right) \tag{26a}$$

$$I_2 = \frac{2}{3}\frac{\ell b}{S}\left(\frac{K_2^3 - K_1^3}{4} + K_2^2 + \frac{3}{2}K_2 + 1\right) \tag{26b}$$

$$I_3 = 2\frac{\ell b}{S}K_1\left(\frac{K_2 - K_1}{3} + \frac{1}{2}\right) \tag{26c}$$

$$I_4 = \frac{\ell b}{S}K_1\left(\frac{K_2^2 - K_1^2}{4} + \frac{2K_2}{3} + \frac{1}{2}\right) \tag{26d}$$

where

$$\frac{S}{\ell b} = K_2 - K_1 + 2, \quad K_1 = \frac{x_1}{\ell}, \quad K_2 = \frac{x_2}{\ell} - 1$$

x_1, x_2 being the abscissas of the wing tip leading and trailing edge points, respectively, as shown in Fig. 8.

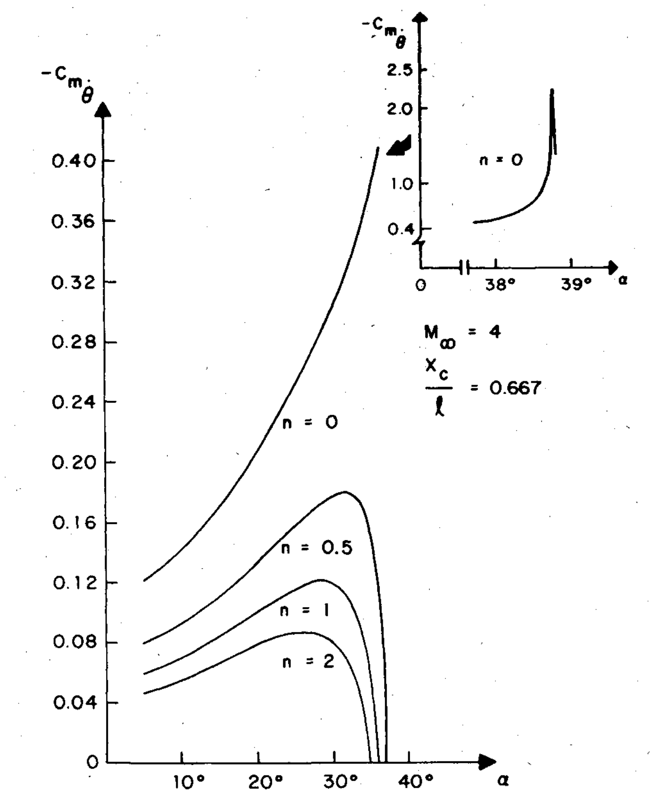

Fig. 7 Variation of damping-in-pitch derivative of power-lawed delta wings with angle of attack.

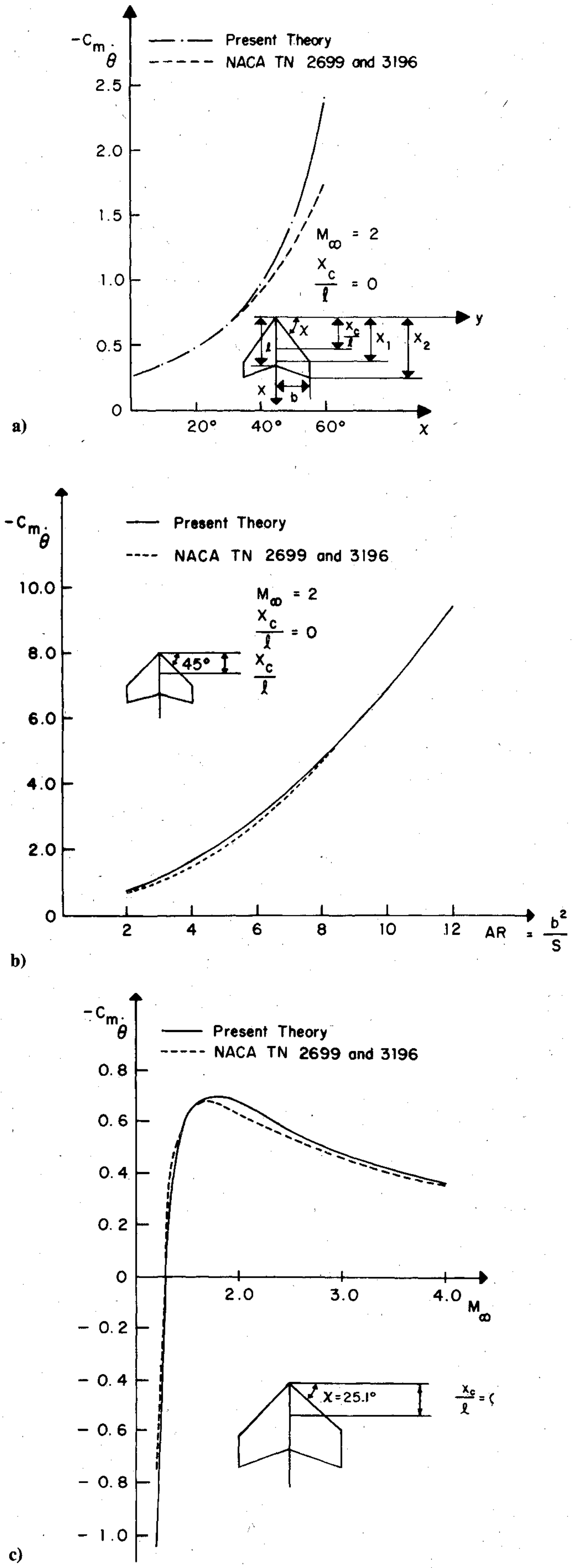

Fig. 8 Comparison of present theory with linearized theory— a) aspect ratio: 3, taper ratio: 0.25; b) 25 deg leading-edge sweep, taper ratio: 0.25; c) 25.1 deg leading-edge sweep, taper ratio: 0.25, aspect ratio: 5.12.

A number of special cases can easily be obtained. For the case of a flat plate, Eqs. (26) reduce to

$$I_1 = 1/2, \quad I_2 = 1/3, \quad I_3 = I_4 = 0, \quad S/\ell b = 2$$

The stability derivatives are then given by

$$-C_{m_\theta} = A\left(\frac{1}{2} - \frac{x_c}{\ell}\right) \tag{27a}$$

$$-C_{m_{\dot\theta}} = \frac{B}{3} - \frac{B+C}{2}\left(\frac{x_c}{\ell}\right) + C\left(\frac{x_c}{\ell}\right)^2 \tag{27b}$$

For the case of a triangular delta wing, $K_1 = 1$, $K_2 = 0$ and Eqs. (26) reduce to

$$I_1 = 2/3, \quad I_2 = 1/2, \quad I_3 = 1/3, \quad I_4 = 1/4, \quad S/\ell b = 1$$

The stability derivatives in this case are given by

$$-C_{m_\theta} = A\left(\frac{2}{3} - \frac{x_c}{\ell}\right) \tag{28a}$$

$$-C_{m_{\dot\theta}} = \frac{B+C}{4} - \frac{B+3C}{3}\left(\frac{x_c}{\ell}\right) + C\left(\frac{x_c}{\ell}\right)^2 \tag{28b}$$

For vanishingly small mean angle of attack α the coefficients A, B, C appearing in Eqs. (14) become

$$A = C = \frac{4}{\beta}; \quad B = 4\frac{\beta^2 - 1}{\beta^3}; \quad \beta = \sqrt{M_\infty^2 - 1}$$

and Eqs. (27) and (28) reduce to the well-known potential flow theory (Ref. 8, pp. 52 and 144). Also note that the stability derivatives of rectangular wings are the same as the flat plate derivatives because of the strip theory assumption.

Comparisons of the present theory with potential flow theory[9,10] for swept-back wings of general planform exhibit good agreement over a wide range of parameter variations. Figure 8a shows the variation of the pitch damping as a function of leading-edge sweep angle for a wing of aspect ratio 3 and taper ratio 0.25; Fig. 8b the variation with aspect ratio for a wing of 25 deg leading-edge sweep and 0.25 taper ratio; Fig. 8c the variation with Mach number for a wing of 25.1 deg leading-edge sweep, 0.25 taper ratio, and 5.12 aspect ratio. Further details can be found in Ref. 11.

VII. Concluding Remarks

In this paper we have used the known exact unsteady flow solutions of Hui over an oscillating flat plate and the strip theory to derive formulas for the stability derivatives of a three-dimensional flat plate wing in supersonic/hypersonic flow. These analytic formulas are in very simple closed form and hold for wings of general planform at arbitrary angles of attack provided the shock wave is attached to the leading edges. They are shown to be in good agreement with existing theories for various special cases ranging from low supersonic to hypersonic flow. They become exact in the Newtonian limit.

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